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AN ALGEBRAIC TURBULENCE MODEL FOR TURBOMACHINERY

by

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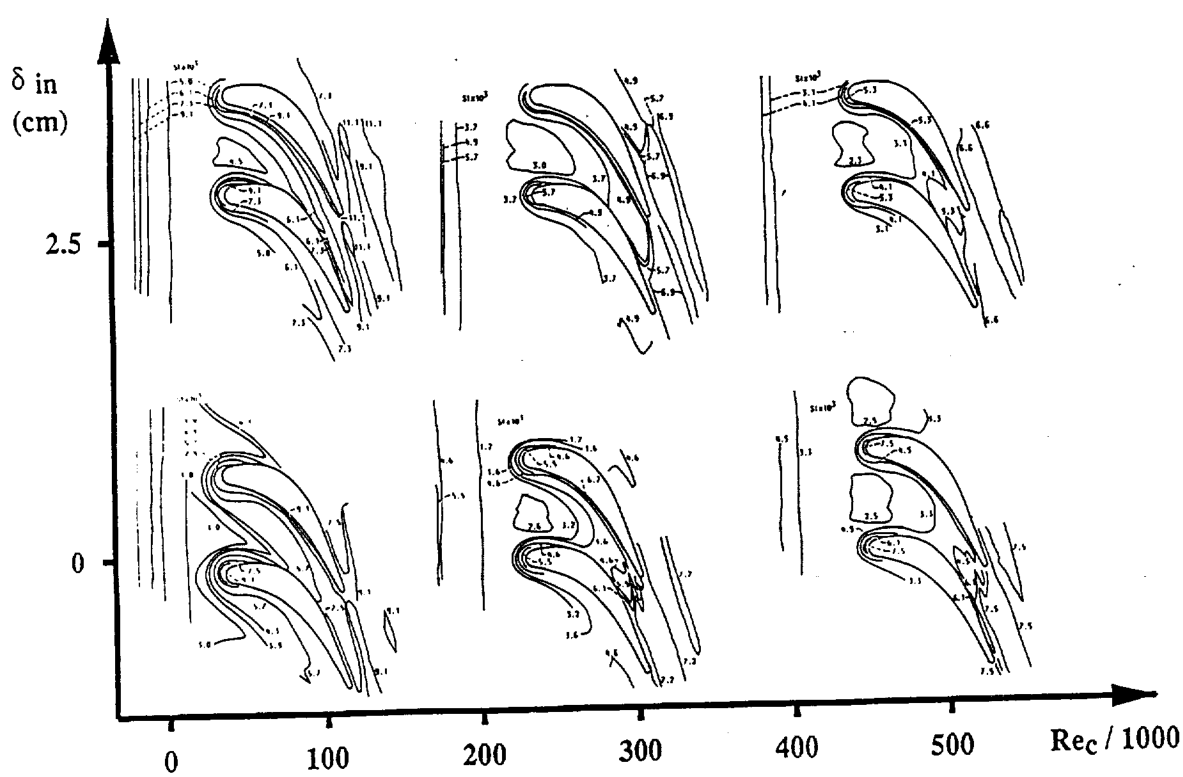
OVERVIEW

- MOTIVATION - TURBINE ENDWALL HEAT TRANSFER
- DESCRIPTION OF NEW MODEL
- RESULTS
  1. FLAT PLATE
  2. ANNULAR TURBINE CASCADE
  3. TURBINE ENDWALL HEAT TRANSFER
  4. SUPERSONIC COMPRESSOR BLADE
- SUMMARY



EXPERIMENTAL ENDWALL STANTON NUMBER CONTOURS AS A FUNCTION OF δ_{inlet} AND Re_{chord}

RVC3D (ROTOR VISCOUS CODE 3-D)

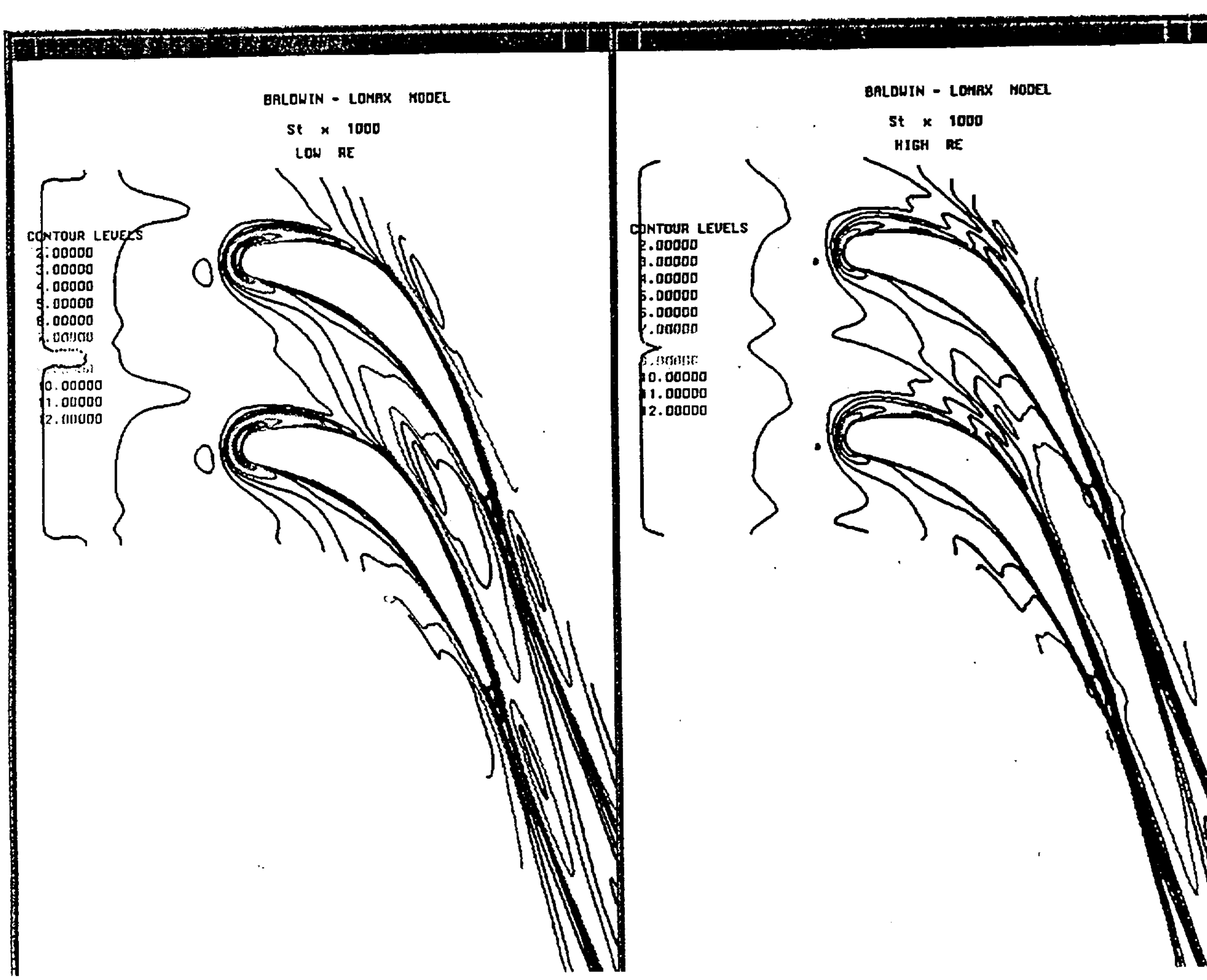
BY R. V. CHIMA

DESCRIPTION

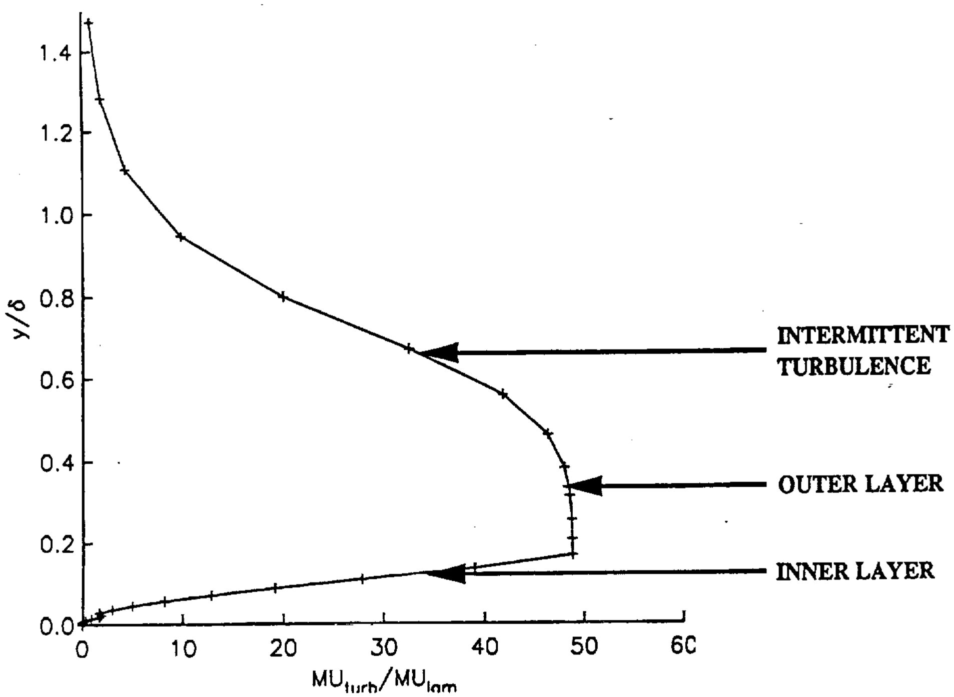
- EULER OR NAVIER-STOKES ANALYSIS
  FOR STEADY 3-D FLOWS IN TURBOMACHINERY

FEATURES

- CARTESIAN FORMULATION, ROTATION ABOUT X-AXIS
  RECTANGULAR OR ANNULAR GEOMETRIES
- SOLVES NAVIER-STOKES EQUATIONS
  THIN-LAYER FORMULATION, (NO STREAMWISE VISCOUS TERMS)
  RETAINS HUB-TO-TIP & BLADE-TO-BLADE VISCOUS TERMS
  BALDWIN-LOMAX OR CEBECI-SMITH TURBULENCE MODEL
  SIMPLE TIP CLEARANCE MODEL
- NODE-CENTERED FINITE-DIFFERENCE FORMULATION
  EXPLICIT 4-STAGE RUNGE-KUTTA TIME-MARCHING SCHEME
  2ND + 4TH ORDER ARTIFICIAL VISCOSITY, EIGENVALUE SCALING
  VARIABLE $\Delta t_{i,j}$ & IMPLICIT RESIDUAL SMOOTHING
  HIGHLY VECTORIZED & AUTOTASKED FOR CRAY Y-MP
- STACKED C-TYPE GRIDS



TURBULENT VISCOSITY PROFILE



CEBECI-SMITH & BALDWIN-LOMAX MODELS

INNER LAYER: PRANDTL-VAN DRIEST FORMULATION

CEBECI-SMITH

$$\begin{aligned} \mu_i &= \rho l^2 |\partial u/\partial y| \\ l &= \kappa y D \\ D &= 1-\exp(-y^+/A^+) \quad \text{VAN DRIEST DAMPING} \end{aligned}$$

BALDWIN-LOMAX

$$\mu_i = \rho l^2 |\omega|$$

OUTER LAYER: CLAUSER FORMULATION

CEBECI-SMITH

$$\begin{aligned} \mu_o &= K\rho\gamma\delta^* u_e \\ \gamma &= \left[1+5.5\left(\frac{y}{\delta}\right)^6\right]^{-1} \quad \text{KLEBANOFF INTERMITTENCY FUNCTION} \end{aligned}$$

BALDWIN-LOMAX

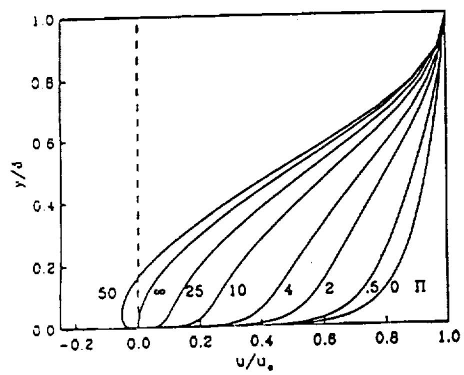
$$\mu_o = K\rho\gamma C_{cp} \min \begin{cases} y_{max} f_{max} \\ \text{wake option} \end{cases}$$

$$f(y) = y|\omega| D$$

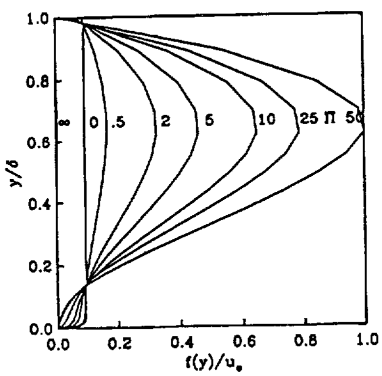
BALDWIN-LOMAX MODEL ANALYSIS

(SEE PAPER FOR DETAILS)

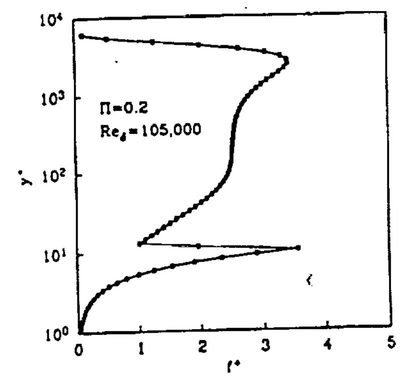
1. ASSUME SUBLAYER–WALL–WAKE VELOCITY PROFILE
2. CALCULATE BALDWIN-LOMAX FUNCTION $f(y)$
   MAX. OCCURS AT $y_{max} = .646\delta$
   INDEPENDENT OF PRESURE GRADIENT
   NO MAX. FOR INFINITELY FAVORABLE $\partial p/\partial x$
3. SPURIOUS MAX. CAN OCCUR AT EDGE OF VISCOUS SUBLAYER
   MOST LIKELY AT LOW Re & FAVORABLE $\partial p/\partial x$



1. VELOCITY PROFILES, $Re_\delta = 105,000$



2. B-L FUNCTION $f(y)$



3. SPURIOUS MAXIMUM IN $f(y)$

PROPOSED TURBULENCE MODEL

INNER LAYER (SIMILAR TO BALDWIN-LOMAX)

$$
\begin{aligned}
\mu_i &= \rho l^2 |\omega| \\
l &= \kappa y D \\
D &= 1 - \exp(-y^+/A^+) \\
y^+ &= y\frac{u^*}{\nu} \\
u^* &= \sqrt{\frac{\tau_{wall}}{\rho}}
\end{aligned}
$$

PROPOSED TURBULENCE MODEL

PRESSURE GRADIENT EFFECTS

- ACCELERATING FLOWS TEND TO RELAMINARIZE
- MODELLED BY INCREASING A^+ IN FAVORABLE $\partial p/\partial s$
- CEBECI'S EXPRESSION FOR A^+ USED:

$$A^+ = \frac{26}{\sqrt{1 + 11.8p^+}}$$

$$p^+ = \frac{\nu}{\rho u^{*3}} \frac{\partial p}{\partial s}$$

- PRESSURE GRADIENT EVALUATED USING:

$$\frac{\partial p}{\partial s} \approx \frac{\vec{V}_e}{|\vec{V}_e|} \cdot \nabla p$$

- "EDGE VELOCITY" $\vec{V}_e$ EVALUATED AT A GRID LINE FAR ENOUGH FROM THE WALL TO GIVE THE GENERAL FLOW DIRECTION
- KAYS-MOFFATT EXPRESSION WAS TESTED, EFFECTS TOO STRONG

PROPOSED TURBULENCE MODEL

LOCAL SHEAR MODEL

- IN STRONGLY ACCELERATING FLOWS τ^+ DECREASES WITH y^+
- MODELLED BY REPLACING τ_{wall} WITH $\tau(y)$ IN D

$$D = 1 - \exp(-y^+/A^+)$$

$$y^+ = y\sqrt{\frac{\rho}{\mu_l} \frac{(\mu_l + \mu_t)}{\mu_l} |\omega|}$$

- ERROR IN ORIGINAL PAPER - USED $\mu_l|\omega|$ ONLY
- USED BY KAYS, PATANKAR-SPALDING, OTHERS
- ALSO USED TO AVOID PROBLEMS AT SEPARATION WHEN $\tau_{wall} \to 0$

PROPOSED TURBULENCE MODEL

OUTER LAYER

$$\begin{aligned}
\mu_o &= K\rho\gamma \min\left\{ \begin{array}{l} F \\ C_{wk}\bar{y}(|V_{max}| - |V_{min}|) \end{array} \right. \\
\gamma &= \left[1 + 5.5\left(\frac{C_{Kleb}\, y}{\bar{y}}\right)^6\right]^{-1} \\
C_{wk} &= 0.825 \\
C_{Kleb} &= 0.55
\end{aligned}$$

PROPOSED TURBULENCE MODEL

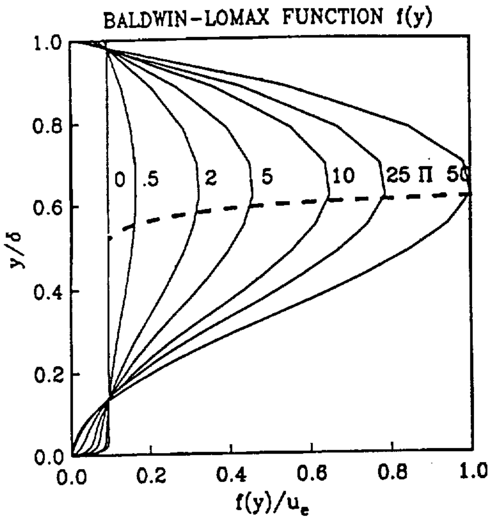
OUTER LAYER - FUNCTION F

- DEFINE $F = \int f dy$
- INTEGRATE BY PARTS ASSUMING $|\omega| \rightarrow 0$ AS $y \rightarrow \delta$

$$\begin{aligned}
F &= \int_0^\infty y|\omega| dy \\
&\approx \int_0^\delta y \frac{\partial u}{\partial y} dy \\
&= uy\Big|_0^\delta - \int_0^\delta u dy \\
&= \int_0^\delta (u_e - u) dy \\
F &= \delta^* u_e
\end{aligned}$$

- USE F DIRECTLY IN CEBECI-SMITH OUTER FORMULATION
- ELIMINATES CONSTANT C_{cp}
- DOES NOT REQUIRE KNOWLEDGE OF δ OR u_e
- DISCOVERED INDEPENDENTLY BY D. A. JOHNSON, AIAA 92-0026

PROPOSED TURBULENCE MODEL



OUTER LAYER - LENGTH SCALE $\bar{y}$

- $\bar{y}$ IS THE CENTROID OF THE $f(y)$ CURVE

$$\int_0^{\bar{y}} f(y)dy = \int_{\bar{y}}^{\delta} f(y)dy$$

- EVALUATE USING COLE'S VELOCITY PROFILES

Π	$\bar{y}/\delta$
0	.5
.5	.55
∞	.606

- USE EQUILIBRIUM VALUE $C_{Kleb} = \bar{y}/\delta = .55$

PROPOSED TURBULENCE MODEL

OUTER LAYER - WAKE MODEL

$$\mu_o = K\rho\gamma \min \begin{cases} F \\ C_{wk}\bar{y}(|V_{max}| - |V_{min}|) \end{cases}$$

- LOWER OPTION IS A CONVENTIONAL WAKE MODEL
- EVALUATE C_{wk} BY EQUATING TWO OPTIONS, ASSUMING

$$\begin{aligned} \bar{y}_{sep} &= .606\,\delta \\ F_{sep} &= u_e\delta/2 \\ \Delta V/u_e &\approx 1 \end{aligned}$$

- GIVES $C_{wk} = 0.825$

PROPOSED TURBULENCE MODEL

3-D IMPLEMENTATION

- GRANVILLE BLENDING FUNCTION

$$\mu_{eff} = \mu_o \tanh \frac{\mu_i}{\mu_o}$$

- MODEL APPLIED INDEPENDENTLY IN BLADE-TO-BLADE (η) AND SPANWISE (ζ) DIRECTIONS
- INNER LAYER - USE BULEEV LENGTH SCALE

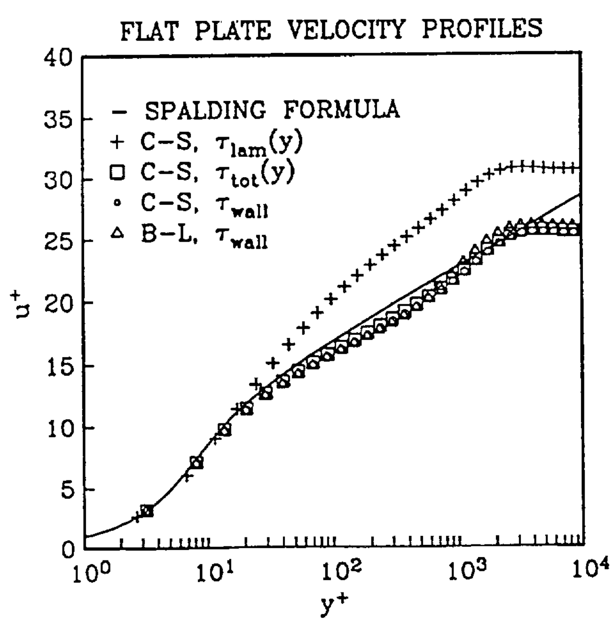
$$y_i = \frac{2 s_\eta s_\zeta}{s_\eta + s_\zeta + \sqrt{s_\eta^2 + s_\zeta^2}}$$

- OUTER LAYER - USE ACTUAL DISTANCE ACROSS PROFILE

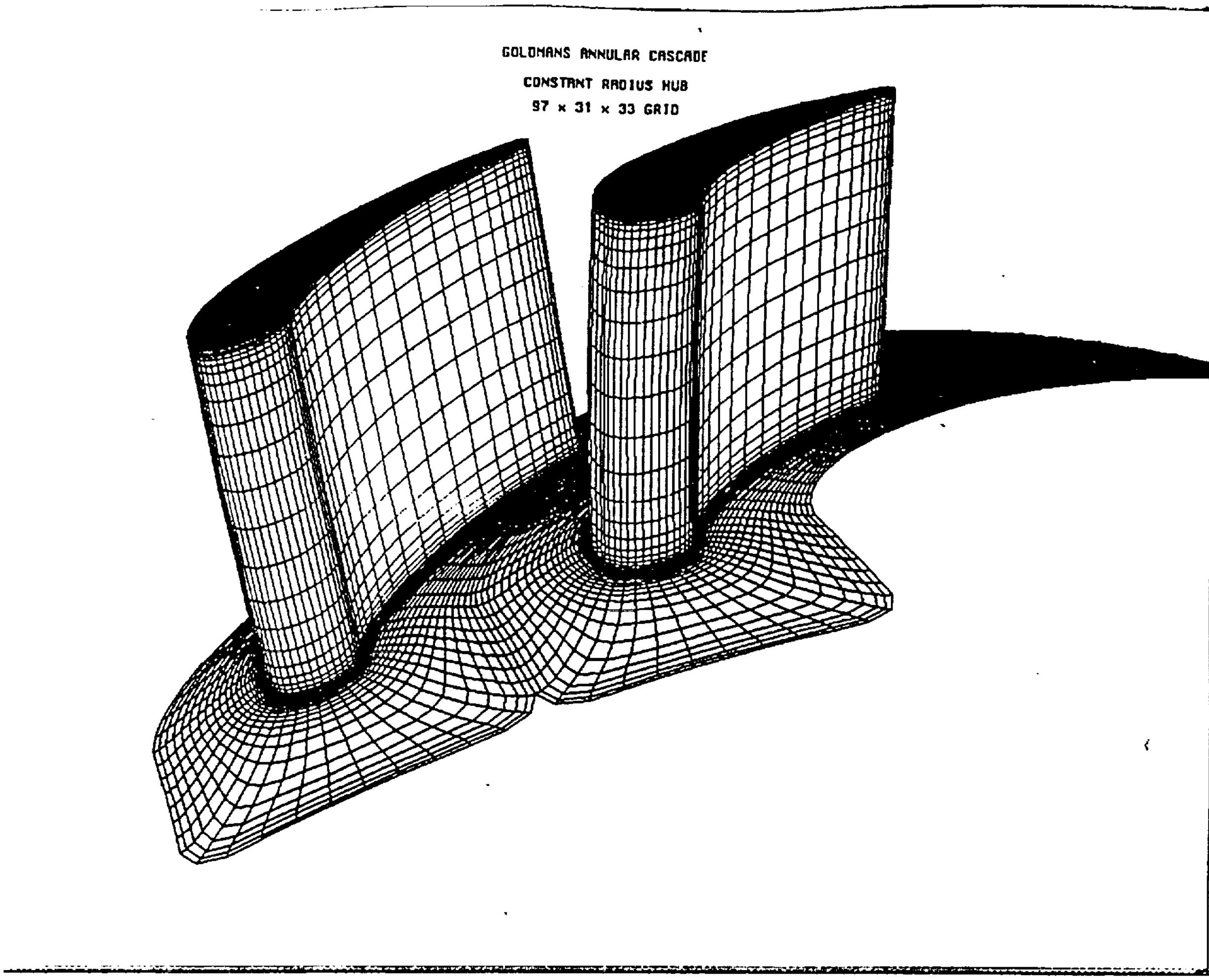
$$y_o = s_\eta \text{ OR } s_\zeta$$

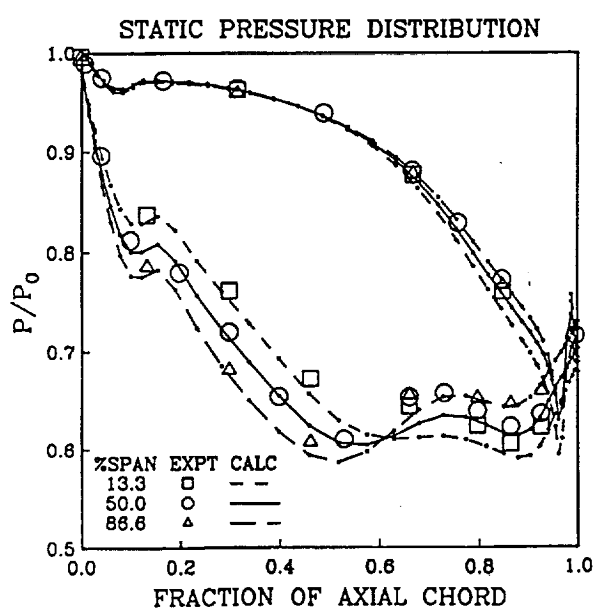
- BLEND η AND ζ PROFILES VECTORALLY

$$\mu_{turb} = \sqrt{\mu_{t\eta}^2 + \mu_{t\zeta}^2}$$

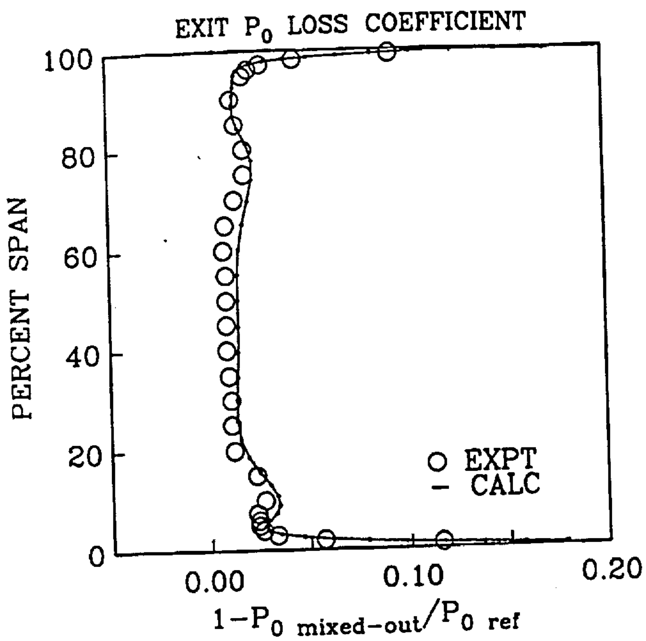


COMPARISON OF FLAT PLATE VELOCITY PROFILES TO SPALDING'S COMPOSITE LAW OF THE WALL

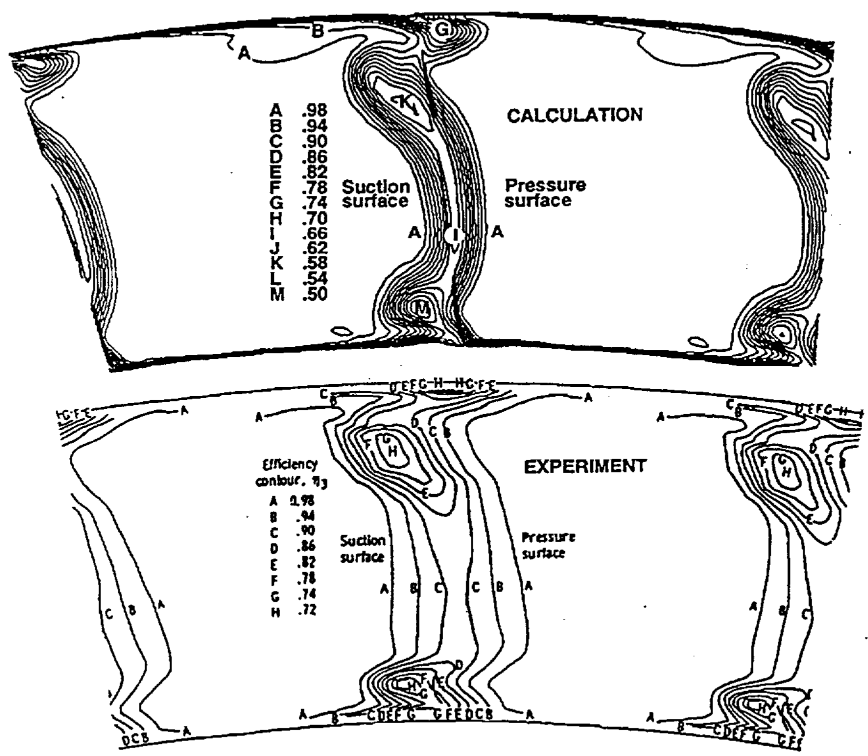




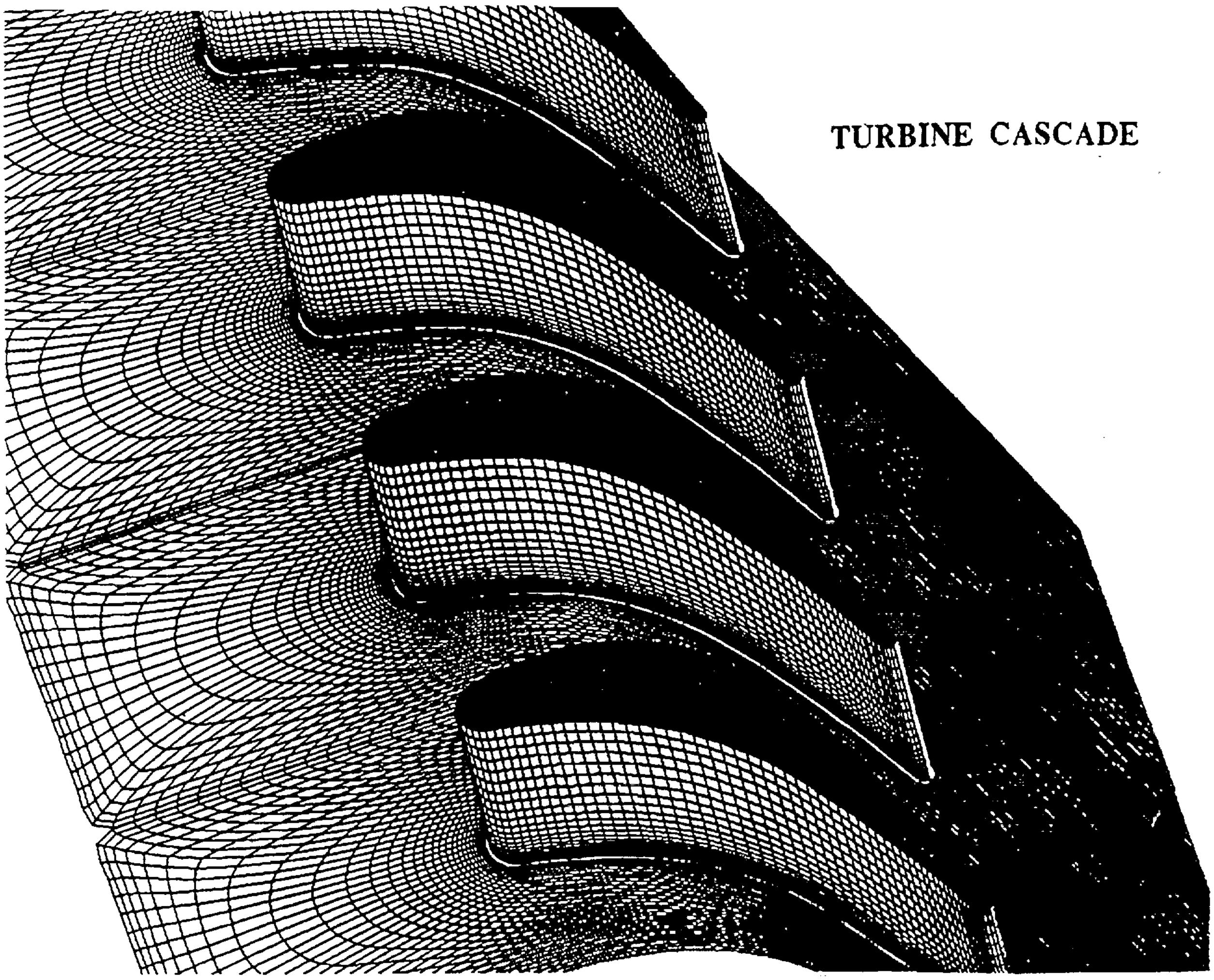
COMPUTED & MEASURED PRESSURE DISTRIBUTIONS
FOR THE ANNULAR TURBINE CASCADE



COMPUTED & MEASURED LOSS COEFFICIENT PROFILES
FOR THE ANNULAR TURBINE CASCADE

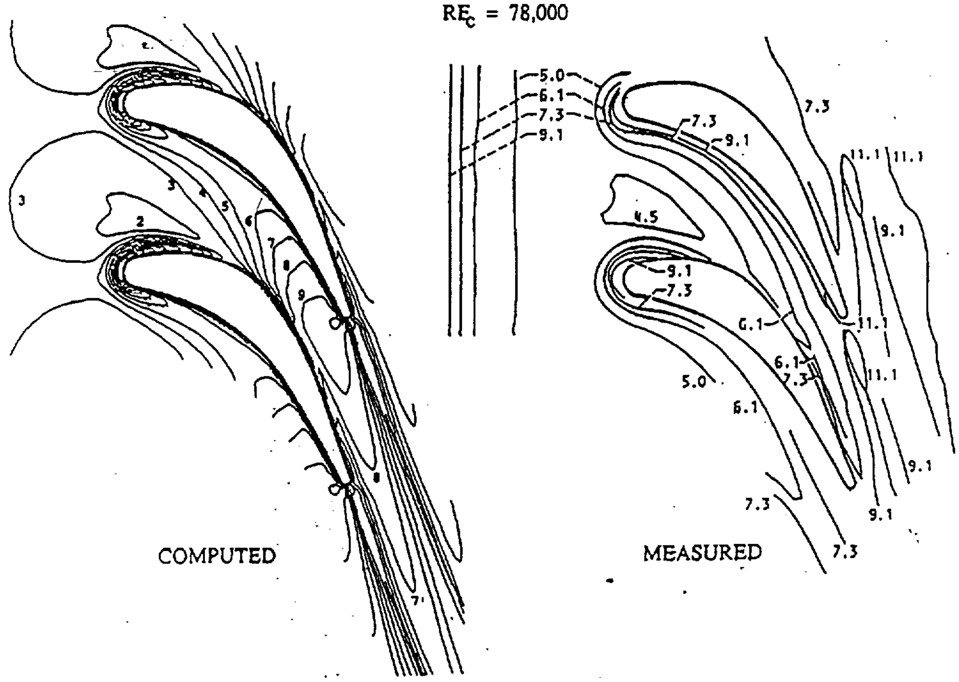


COMPUTED & MEASURED EFFICIENCY CONTOURS
IN THE WAKE OF THE ANNULAR TURBINE CASCADE

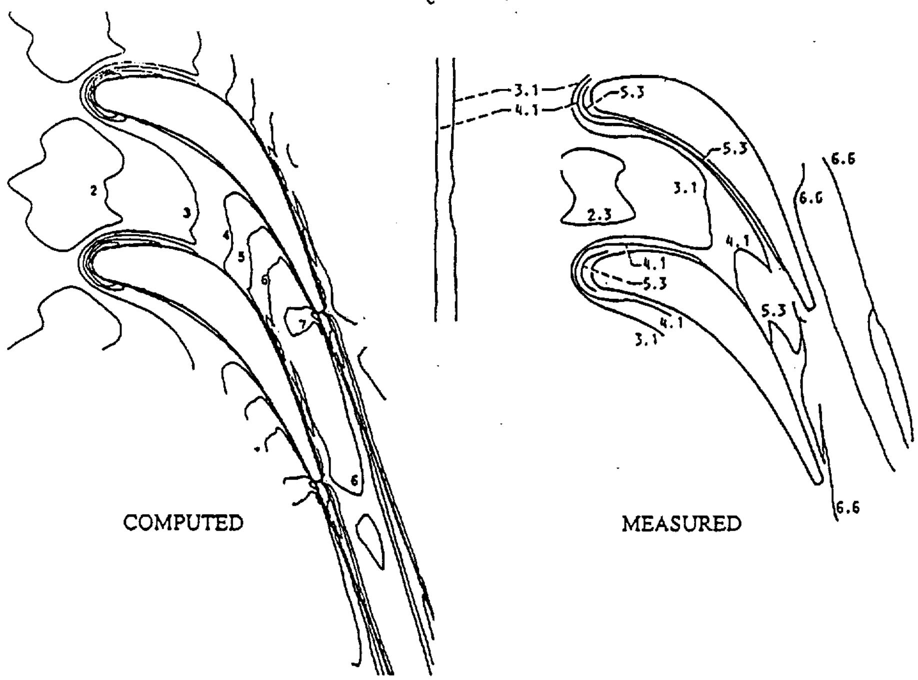


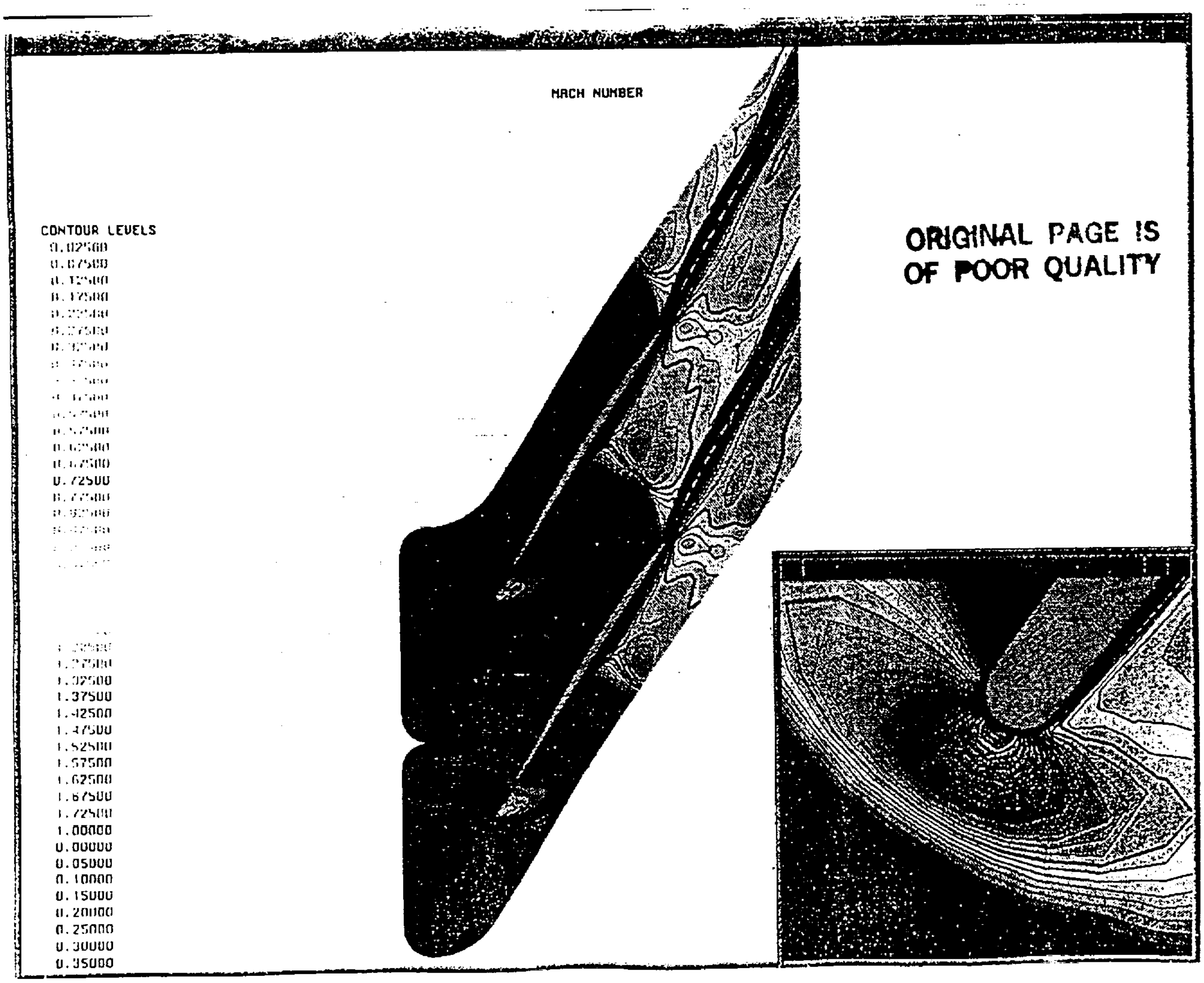
TURBINE CASCADE

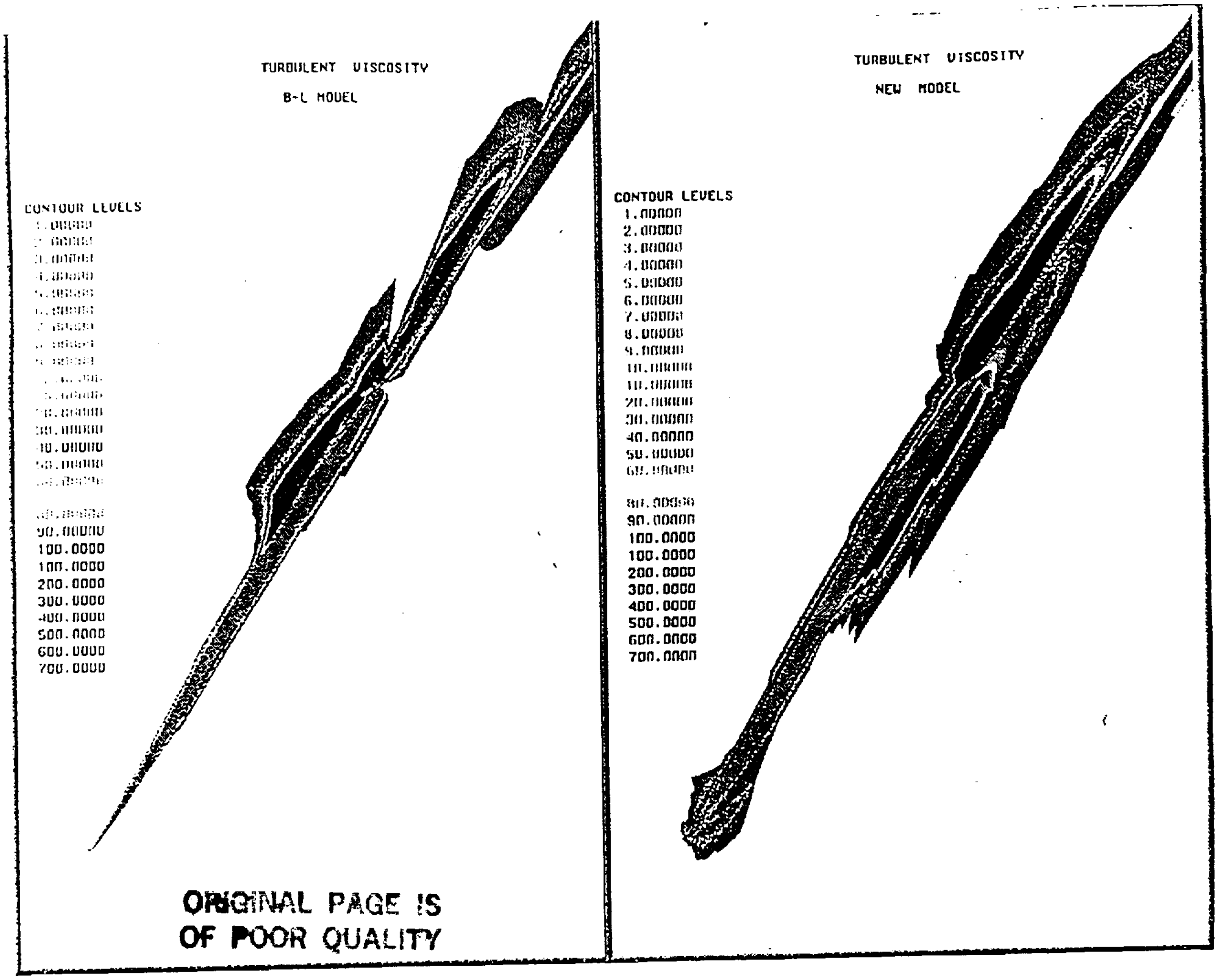
BOYLE'S LINEAR CASCADE
STANTON NUMBER x 1000
RE_C = 78,000



BOYLE'S LINEAR CASCADE
STANTON NUMBER x 1000
RE_c = 490,000







SUMMARY

- SPURIOUS MAXIMUM IN B-L FUNCTION $f(y)$ CAN GIVE INCORRECT TURBULENT LENGTH SCALE & ERRATIC St OR C_f PATTERNS
  - MOST LIKELY AT LOW Re AND FAVORABLE $\partial p/\partial s$
- NEW TURBULENCE MODEL PROPOSED
  - INTEGRAL RELATIONS FOR $\delta^* u_e$ AND δ USED WITH C-S MODEL
  - EFFECTS OF $\partial p/\partial s$ MODELED
  - WAKE MODEL PROPOSED
- FLAT PLATE
  - B-L & NEW MODEL AGREE WITH LAW OF THE WALL
  - LOCAL SHEAR MOD. DOES NOT AGREE WITH LAW OF THE WALL
- ANNULAR TURBINE
  - GOOD AGREEMENT WITH EXPT. PRESSURE DISTRIBUTION
  - WAKE MIXING UNDER-PREDICTED
- TURBINE ENDWALL HEAT TRANSFER
  - VARIATIONS IN ENDWALL St WITH Re PREDICTED WELL
  - EFFECTS OF $\partial p/\partial s$ IMPORTANT
- TRANSONIC FAN
  - SHEAR LAYER FROM BOW SHOCK ACTS LIKE VISCOUS LAYER
  - NEW MODEL OVERPREDICTS L.E. μ_t
  - B-L MODEL PREDICTS REASONABLE L.E. μ_t